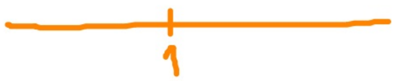
Rovnice a nerovnice

1) $x^2+2|x-1|-6=0$



a) pro $x \in (-\infty; 1\rangle$

$x^2+2\cdot(1-x)-6=0$

$x^2-2x-4=0$

$x_{1,2}=\frac{2\pm\sqrt{4+16}}{2}=\frac{2\pm2\sqrt{5}}{2}$

$x_1=1-\sqrt{5}$

$x_2=1+\sqrt{5}$

$K_1=\{1-\sqrt{5}\}$

b) pro $x \in \langle 1; \infty)$

$x^2+2(x-1)-6=0$

$x^2+2x-8=0$

$$\left.\begin{array}{l} x_1+x_2=-2 \\ x_1\cdot x_2=-8 \end{array}\right\} \begin{array}{l} x_1=-4 \\ x_2=2 \end{array}$$

$K_2=\{2\}$

$$\underline{K=\{1-\sqrt{5}; 2\}}$$

2) $\sqrt{1+x} - \sqrt{4-x} = 1$

$1+x \geqq 0 \Rightarrow x \geqq -1$

$4-x \geqq 0 \Rightarrow x \leqq 4$

$D = \langle -1; 4 \rangle$

$\sqrt{1+x} = 1 + \sqrt{4-x}$

$1+x = 1 + 2\sqrt{4-x} + (4-x)$

$2x - 4 = 2\sqrt{4-x}$

$x - 2 = \sqrt{4-x}$

$x^2 - 4x + 4 = 4 - x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x_1 = 0$

$x_2 = 3$

Zk:

a) pro $x = 0$

$L_1 = \sqrt{1+0} - \sqrt{4-0} = -1$

$P_1 = 1$ $\qquad L_1 \neq P_1$

b) pro $x = 3$

$L_2 = \sqrt{1+3} - \sqrt{4-3} = 1$

$P_2 = 1$ $\qquad L_2 = P_2$

$\underline{K = \{3\}}$

3)

$$x+1-\frac{2x+a+1}{a}=\frac{a-x}{a}$$

$\boxed{a\in\mathbb{R}}$

a) pro $a=0$ nemá smysl

b) pro $a\neq 0$

$$ax+a-(2x+a+1)=a-x$$
$$ax+a-2x-a-1=a-x$$
$$ax-x=a+1$$
$$x(a-1)=a+1$$

pro $a=1$

$$0=2$$
$$K_1=\emptyset$$

pro $a\neq 1$

$$x=\frac{a+1}{a-1}$$
$$K_2=\left\{\frac{a+1}{a-1}\right\}$$

pro $a=0$ nemá smysl

pro $a=1$ je $K=\emptyset$

pro $a\neq 0 \wedge a\neq 1$ je $K=\left\{\frac{a+1}{a-1}\right\}$

$\boxed{D.C.}$

1) $|3x-1| - |2x+3| = 0$ $\qquad K = \{-\frac{2}{5} ; 4\}$

2) $|x+1| + |2-x| - |x+3| = 4$ $\qquad K = \{-2 ; 8\}$

3) $|x^2+2x-3| = 3-2x-x^2$

4) $(x+2)(m-1) = 3mx \quad ; \quad m \in \mathbb{R}$

5) $m^2 x = m(x+2) - 2 \quad ; \quad m \in \mathbb{R}$

6) $(x+3)(1-c^2) = x + x \cdot c^3 \quad ; \quad c \in \mathbb{Z}$

7) $\sqrt{2x-1} + \sqrt{3x+1} = 3$ $\qquad K = \{1\}$

8) $\sqrt{2x+1} + \sqrt{x-3} = 2\sqrt{x}$ $\qquad K = \{4\}$

9) $\sqrt{1-x} = \sqrt{6-x} - \sqrt{-5-2x}$ $\qquad K = \{-3\}$

$$\begin{aligned} x + 2y - z &= 2 \quad (1) \\ 2x + y + z &= 7 \quad (2) \\ x + y + z &= 6 \quad (3) \end{aligned}$$

$$\begin{aligned} (1):&\quad x + 2y - z = 2 \quad (1) \\ (2)-2(1):&\quad -3y + 3z = 3 \quad (2) \\ (3)-(1):&\quad -y + 2z = 4 \quad (3) \end{aligned}$$

$$\begin{aligned} (1):&\quad x + 2y - z = 2 \Rightarrow x = 1 \\ -1\cdot(3):&\quad y - 2z = -4 \Rightarrow y = 2 \\ (2)-3(3):&\quad -3z = -9 \Rightarrow z = 3 \end{aligned}$$

$$K = \{[1;2;3]\}$$

$$
\begin{aligned}
3x + 4y + z + 2u &= -3 \quad (1)\\
3x + 5y + 3z + 5u &= -6 \quad (2)\\
6x + 8y + z + 5u &= -8 \quad (3)\\
3x + 5y + 3z + 7u &= -8 \quad (4)
\end{aligned}
$$

$$
\begin{aligned}
(1):&\quad 3x+4y+z+2u = -3 &\Rightarrow x = 2\\
(2)-(1):&\quad y+2z+3u = -3 &\Rightarrow y = -2\\
(3)-2(1):&\quad -z+u = -2 &\Rightarrow z = 1\\
(4)-(2):&\quad 2u = -2 &\Rightarrow u = -1
\end{aligned}
$$

$$K = \{[2; -2; 1; -1]\}$$

$$x + (p-1)y = 1 \quad \Rightarrow x = 1-(p-1)y$$
$$(p+1)\cdot x + 3y = -1$$

$p \in \mathbb{R}$

$$(p+1)\cdot\left[1-(p-1)\cdot y\right] + 3y = -1$$
$$(p+1)\cdot(1-py+y) + 3y = -1$$
$$p - p^2y + py + 1 - py + y + 3y = -1$$
$$y(-p^2+4) = -2-p$$
$$y(p^2-4) = p+2$$
$$y(p+2)(p-2) = p+2$$

a) pro $p = -2$

$$0 = 0$$

$$K_1 = \left\{\left[1+3y;\, y\right];\, y \in \mathbb{R}\right\}$$

b) pro $p = 2$

$$0 = 4$$
$$K_2 = \emptyset$$

c) pro $p \neq -2 \wedge p \neq 2$

$$y = \frac{1}{p-2}$$
$$x = 1 - \frac{p-1}{p-2} = \frac{p-2-p+1}{p-2} = \frac{-1}{p-2}$$
$$K_3 = \left\{\left[\frac{-1}{p-2};\, \frac{1}{p-2}\right]\right\}$$

$$px + y = p \quad \Rightarrow y = p - px$$
$$(p+2)x + py = 4$$

$p \in \mathbb{R}$

$$(p+2)x + p(p-px) = 4$$
$$px + 2x + p^2 - p^2x - 4 = 0$$
$$x(-p^2+p+2) = 4-p^2$$
$$x(p^2-p-2) = p^2-4$$
$$x \cdot (p-2)(p+1) = (p+2)(p-2)$$

a) pro $p=-1$

$0 = -3$

$K_1 = \emptyset$

b) pro $p=2$

$0=0$

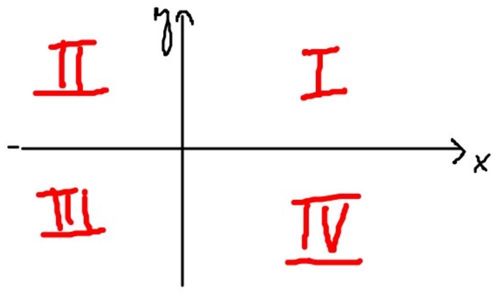
$K_2 = \{[x; 2-2x]; x \in \mathbb{R}\}$

c) pro $p \neq -1 \wedge p \neq 2$

$$x = \frac{p+2}{p+1}$$
$$y = p - p\frac{p+2}{p+1} = \frac{p^2+p-p^2-2p}{p+1} = \frac{-p}{p+1}$$
$$K_3 = \left\{\left[\frac{p+2}{p+1}; \frac{-p}{p+1}\right]\right\}$$

$$2x + 3y = p$$
$$x - y = 1$$

řešení ve III. kvadrantu



$$x = 1 + y$$
$$2(1+y) + 3y = p$$
$$2 + 2y + 3y = p$$
$$5y = p - 2$$
$$y = \frac{p-2}{5}$$

$$x = 1 + y = 1 + \frac{p-2}{5}$$
$$x = \frac{5+p-2}{5} = \frac{p+3}{5}$$

III.: $x \leqq 0 \qquad y \leqq 0$

$$\frac{p-2}{5} \leqq 0 \qquad \frac{p+3}{5} \leqq 0$$

$$\underbrace{p \leqq 2 \qquad p \leqq -3}$$

$$p \in (-\infty; -3\rangle$$

$$x^2+2x+m^2=0 \qquad m\in\mathbb{R}$$

$$D=4-4m^2=4(1-m^2)=4(1-m)(1+m)$$

$$\begin{array}{ccccc} - & & + & & - \\ \hline & -1 & & 1 & \end{array}$$

a) pro $m\in(-\infty;-1)\cup(1;\infty)$

$$D<0 \Rightarrow K_1=\emptyset$$

b) pro $m\in\{-1;1\}$

$$D=0 \Rightarrow x=\frac{-2}{2}=-1 \Rightarrow K_2=\{-1\}$$

c) pro $m\in(-1;1)$

$$D>0 \Rightarrow x_{1|2}=\frac{-2\pm\sqrt{4-4m^2}}{2}=-1\pm\sqrt{1-m^2} \Rightarrow K_3=\left\{\begin{matrix}-1-\sqrt{1-m^2};\\ -1+\sqrt{1-m^2}\end{matrix}\right\}$$

$x^2+4x-3m^2+7m-2=0 \qquad m\in\mathbb{R}$

jeden kořen rovný nule

určete druhý kořen

$-3m^2+7m-2=0$

$3m^2-7m+2=0$

$m_{1,2}=\frac{7\pm\sqrt{49-24}}{6}$ $m_1=\frac{1}{3}$, $m_2=2$

a) pro $m=\frac{1}{3}$

$x^2+4x-\frac{1}{3}+\frac{7}{3}-2=0$

$3x^2+12x=0$

$3x(x+4)=0 \Rightarrow \underline{x_2=-4}$

b) pro $m=2$

$x^2+4x-12+14-2=0$

$x^2+4x=0$

$\underline{x_2=-4}$

Lineární a kvadratické nerovnice

1) $|2x-3| \geqq |3x-2|$

a) pro $x \in (-\infty; \frac{2}{3}\rangle$

$-2x+3 \geqq -3x+2$

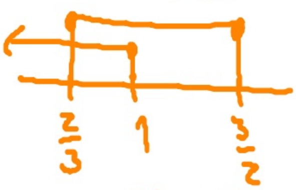
$x \geqq -1$

$K_1 = \langle -1; \frac{2}{3}\rangle$

b) pro $x \in \langle \frac{2}{3}; \frac{3}{2}\rangle$

$-2x+3 \geqq 3x-2$

$1 \geqq x$

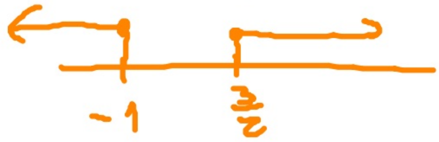


$K_2 = \langle \frac{2}{3}; 1\rangle$

c) pro $x \in \langle \frac{3}{2}; \infty)$

$2x-3 \geqq 3x-2$

$-1 \geqq x$



$K_3 = \emptyset$

$K = \langle -1; 1\rangle$

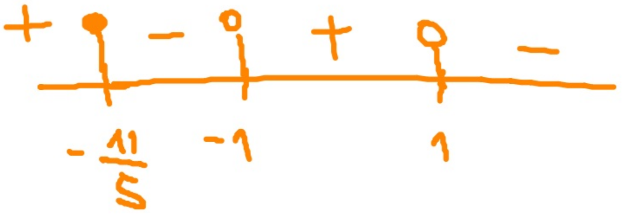
2) $\frac{5-x}{x-1} + \frac{1+4x}{2x+2} \leqq 1$

$$\left.\begin{matrix} x \neq 1 \\ x \neq -1 \end{matrix}\right\} D = \mathbb{R} - \{-1; 1\}$$

$$\frac{(5-x)(2x+2) + (1+4x)(x-1) - (x-1)(2x+2)}{2(x-1)(x+1)} \leqq 0$$

$$\frac{-2x^2+8x+10+4x^2-3x-1-2x^2+2}{2(x-1)(x+1)} \leqq 0$$

$$\frac{5x+11}{2(x-1)(x+1)} \leqq 0$$



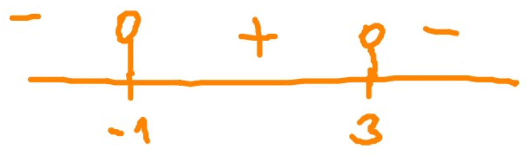
$$K = \left(-\infty; -\frac{11}{5}\right\rangle \cup (-1; 1)$$

3) $x^2-2x-3<0$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x_1=-1 \quad x_2=3$



$K=(-1;3)$

4) $\sqrt{x-2} < x$

$x \geq 2$

$D = \langle 2; \infty)$

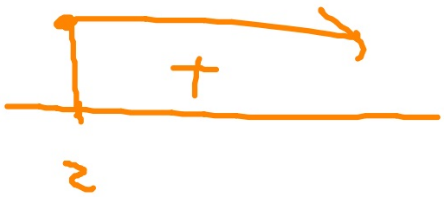
$x-2 < x^2$

$x^2 - x + 2 > 0$

$x^2 - x + 2 = 0$

$x_{1|2} = \frac{1 \pm \sqrt{1-8}}{2}$

$D < 0 \Rightarrow$ rovnice nemá řešení $\Rightarrow$ nejsou nulové body



$\underline{\underline{K = \langle 2; \infty)}}$

5) $$1 \leqq \frac{2\log x + 3}{5} \leqq \frac{5}{2}$$ $\boxed{x > 0}$

a) $$1 \leqq \frac{2\log x + 3}{5}$$

$$5 \leqq 2\log x + 3$$

$$\log x \geqq 1$$

$$x \geqq 10$$

$$K_1 = \langle 10; \infty)$$

b) $$\frac{2\log x + 3}{5} \leqq \frac{5}{2}$$

$$4\log x + 6 \leqq 25$$

$$4\log x \leqq 19$$

$$\log x \leqq \frac{19}{4}$$

$$x \leqq \sqrt[4]{10^{19}}$$

$$K_2 = (0; \sqrt[4]{10^{19}}\rangle$$

$$\underline{K = \langle 10; \sqrt[4]{10^{19}}\rangle}$$

6) $\dfrac{1-2x}{x-1} > -3$ $\qquad (-\infty; 1) \cup (2; \infty)$

7) $\dfrac{x^2+5x+4}{x^2-5x-6} < 0$ $\qquad (-4; -1) \cup (-1; 6)$

8) $2x^2-3x+4 \geqq x^2+2x-2$ $\qquad (-\infty; 2\rangle \cup \langle 3; \infty)$

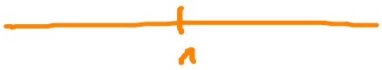
9) $|x| + |x-1| \geqq 2$ $\qquad (-\infty; -\frac{1}{2}\rangle \cup \langle \frac{3}{2}; \infty)$

10) $\dfrac{1}{|x-1|} \geqq \dfrac{2}{|x-2|}$ $\qquad \langle 0; 1) \cup (1; \frac{4}{3}\rangle$

11) $-1 \leqq \dfrac{\log x + 1}{3} \leqq 2$ $\qquad \langle 10^{-4}; 10^{5}\rangle$

12) $\sqrt{x+1} \leq x-1$

$$\left.\begin{array}{l} x+1 \geq 0 \\ x \geq -1 \end{array}\right\} D = \langle -1;\infty)$$



a) $x \in \langle -1;1)$

$$\left.\begin{array}{l} L \geq 0 \\ P < 0 \end{array}\right\}$$ nelze umocnit

$K_1 = \emptyset$

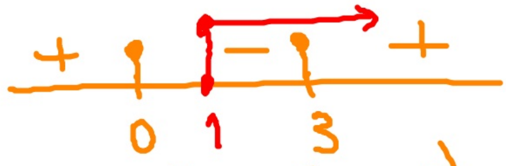
b) $x \in \langle 1;\infty)$

$$\left.\begin{array}{l} L \geq 0 \\ P \geq 0 \end{array}\right\}$$ lze umocnit

$x+1 \leq x^2-2x+1$

$x^2-3x \geq 0$

$x(x-3) \geq 0$



$K_2 = \langle 3;\infty)$

$$\underline{\underline{K = \langle 3;\infty)}}$$

13) $\sqrt{x+1} > x-1$

$\langle -1; 3)$

Exponenciální rovnice a nerovnice

1) $2^{x+1} < 4$

$2^{x+1} < 2^2$

$x+1 < 2$

$x < 1$

$\underline{K = (-\infty; 1)}$

2) $5^{x-4} = 0{,}008$

$5^{x-4} = \frac{1}{125}$

$5^{x-4} = 5^{-3}$

$x - 4 = -3$

$x = 1$

$\underline{K = \{1\}}$

3) $$\frac{10^{x^2}}{2^{-15}} = \frac{5^{-15}}{10^{12-12x}}$$

$$10^{x^2-12x+12} = 10^{-15}$$

$$x^2-12x+12 = -15$$

$$x^2-12x+27 = 0$$

$$(x-3)(x-9) = 0$$

$$x_1 = 3 \quad x_2 = 9$$

$$\underline{\underline{K = \{3; 9\}}}$$

4) $3^{2x-1} + 3^{x} - 3^{0} = 3^{-1}$

$$\frac{(3^x)^2}{3} + 3^x - 1 = \frac{1}{3}$$

$$(3^x)^2 + 3.3^x - 4 = 0$$

subst: $y = 3^x$

$$y^2 + 3y - 4 = 0$$

$$(y+4)(y-1) = 0$$

$y_1 = -4 \Rightarrow 3^x = -4 \Rightarrow K_1 = \emptyset$

$y_2 = +1 \Rightarrow 3^x = 1 \Rightarrow 3^x = 3^0 \Rightarrow x = 0 \Rightarrow K_2 = \{0\}$

$K = \{0\}$

5) Určete souřadnice průsečíku funkcí

$f(x) = 7^{x+1} - 19 \qquad g(x) = 7^{x} + 23$

$$7^{x+1} - 19 = 7^{x} + 23$$
$$7 \cdot 7^{x} - 7^{x} = 42$$
$$7^{x}(7-1) = 42$$
$$7^{x} = 7^{1}$$
$$x = 1$$

$$y = 7^{2} - 19 = 30$$

$$\underline{\underline{P[1;30]}}$$

6) $\left(\frac{4}{25}\right)^{x+3} \cdot \left(\frac{125}{8}\right)^{4x-1} = \frac{5}{2}$ $\{1\}$

7) $\left(\frac{4}{9}\right)^{x} \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{\log 4}{\log 8}$ $\{2\}$

8) $\sqrt[x+\frac{1}{2}]{729} = \sqrt[x-\frac{1}{2}]{9}$ $\{1\}$

9) $5 \cdot 4^{x+1} - 4^{x+2} = 4^{x-1} + 240$ $\{3\}$

10) $4^{x} - 10 \cdot 2^{x-1} = 24$ $\{3\}$

11) $f(x) = 8^{x-1} - 3$ $g(x) = 7 \cdot 8^{x-2} + 5$ $[3;6]$

12) $5^{4x+1}+4>629$ $\left(\frac{3}{4};\infty\right)$

13) $\left(\frac{1}{4}\right)^{\frac{3x^2-1}{2}} \leqq \left(\frac{1}{8}\right)^{\frac{x^2+1}{3}}$ $(-\infty;-1\rangle \cup \langle 1;\infty)$

14) $\frac{1}{4^x}+\frac{1}{2^x} \geqq 20$ $(-\infty;-2\rangle$

Logaritmické rovnice a nerovnice

$$\log_a x = y \iff a^y = x$$

$$x > 0$$
$$a > 0 \wedge a \neq 1$$

1)

$$\log_{\frac{1}{2}}(x-2) \geqq -3$$

$$x - 2 > 0$$
$$x > 2$$
$$D = (2; \infty)$$

$$\log_{\frac{1}{2}}(x-2) \geqq \log_{\frac{1}{2}} 8$$

$$(x-2) \leqq 8$$

$$x \leqq 10$$

$$\underline{K = (2; 10\rangle}$$

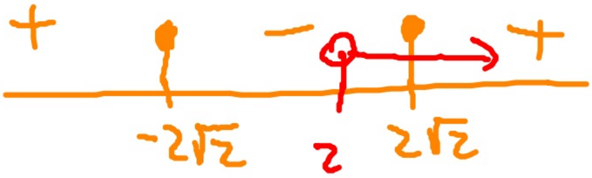
2) $\log_{\frac{1}{2}}(x-2) + \log_{\frac{1}{2}}(x+2) \leqq -2$

$$\log_{\frac{1}{2}}\left[(x-2)\cdot(x+2)\right] \leqq \log_{\frac{1}{2}} 4$$

$$x^2-4 \geqq 4$$

$$x^2-8 \geqq 0$$

$$(x+2\sqrt{2})(x-2\sqrt{2}) \geqq 0$$



$$\underbrace{\begin{array}{l} x-2>0 \Rightarrow x>2 \\ x+2>0 \Rightarrow x>-2 \end{array}}$$

$D = (2; \infty)$

$$\underline{\underline{K = \langle 2\sqrt{2}; \infty)}}$$

3) $$\frac{2\log x}{\log(5x-4)} = 1$$

$$2\log x = \log(5x-4)$$

$$\log x^2 = \log(5x-4)$$

$$x^2 = 5x-4$$

$$x^2-5x+4 = 0$$

$$(x-1)(x-4) = 0$$

$$x_1 = 1 \quad x_2 = 4$$

$x > 0$

$5x-4 > 0 \Rightarrow x > \frac{4}{5}$

$5x-4 \neq 1 \Rightarrow x \neq 1$

$D = \left(\frac{4}{5}; 1\right) \cup (1; \infty)$

$$\underline{K = \{4\}}$$

4) $\log(3x-4)^2 + \log(7x-9)^2 = 2$ $\left\{2; \frac{13}{21}\right\}$

5) $1 + \log x^3 = \frac{10}{\log x}$ $\left\{\sqrt[3]{10^5}; \frac{1}{100}\right\}$

6) $\log_3 x + \log_3(x+1) = 2 - \log_3 \frac{3}{2}$ $\{2\}$

7) $\log\sqrt{x^2-4} - \log\sqrt{x+2} < \log 5$ $(2; 27)$

8) $\log_2(x^2-x-12) < 3$ $(-4; -3) \cup (4; 5)$

9) $\log^2_{\frac{1}{3}} x - 4 > 0$ $\left(0; \frac{1}{9}\right) \cup (9; \infty)$

10) $\dfrac{3 + 2\log x}{3} \leqq 5$ $\qquad (0; 10^6\rangle$

11) $\log_8 (x^2 - 4x + 3) < 1$ $\qquad (-1; 1) \cup (3; 5)$

12) $\log_3^2 x + \log_3 x \geqq 2$ $\qquad (0; \frac{1}{9}\rangle \cup \langle 3; \infty)$

13) $\log_{\frac{1}{3}} \dfrac{3x-1}{x+2} < 1$ $\qquad (-\infty; -2) \cup (\frac{5}{8}; \infty)$

14) $\log 10^{\log(x^2+21)} > 1 + \log x$ $\qquad (0;3) \cup (7; \infty)$

15) $\log_{\frac{1}{2}} (x^2 - 4) > \log_{\frac{1}{2}} (8 - 2x - x^2)$ $\qquad (-3; -2)$